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The objective analysis of some 500 mb. charts.

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F. H. Bushby

(Meteorological Office)

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

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1. Introduction

The recent results obtained from the production of prognostic charts by numerical integration of systems of equations relating to various models of the atmosphere have been very encouraging. Indeed, in America and Sweden forecast charts are already being produced on a routine basis under operational conditions. This has only been made possible by the use of electronic computing machines, and it is natural to consider whether or not it is desirable to use these machines for any of the preliminary work prior to the actual integration calculation.

At present, most if not all of the integration procedures make use of the calculus of finite differences, and it is necessary to know the initial fields of the basic variables at mesh points of a horizontal grid. This is normally done by plotting observations on a map and then drawing isopleths of the required variable. A grid is then superimposed on the map and the initial values of the variable (such as 500 mb. height) are read off from the map at grid points by interpolation by eye from the isopleths. This is a subjective process and Gilchrist and Cressman (1954) and Bergthorrson and Döös (1955) have suggested how this can be done objectively by computation. The advantages of this technique of analysis are that it is objective, and that it is time-saving when fast electronic computing machines are available. A possible disadvantage is that the conventional analyst often puts into his analysis something as a result of his experience (especially where observations are scanty), and it is difficult for an objective technique to make use of this kind of experience.

It was therefore decided to analyse objectively a few 500 mb. charts in order to ascertain whether or not a satisfactory technique could be evolved which would not necessarily be dependent upon a dense network of observations. This report describes the method used and the results of these computations.

2. The method

The technique used by Bergthorrson and Döös (1955) for obtaining contour heights at grid points was essentially an empirical modification of a weighted mean of forecast and climatological data by the observations. That proposed by Gilchrist and Cressman (1954) was the finding of the best quadratic surface to fit the observations over a limited area. In each case the geostrophic approximation was used to enable wind observations to be taken into account.

The method of computing the 500 mb. contour heights which was tested at Dunstable is one which has something in common with both of these techniques. Each grid point is dealt with individually and essentially the method consists of finding, by the method of least squares, the quadratic or cubic surface which

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best fits the observed 500 mb. contour heights and winds and the forecast 500 mb. contour heights. Different weights are given to observed heights and winds and to forecast heights. The method differs from that of Gilchrist and Cressman in that it makes use of forecast values and it differs from the method of Bergthorrson and Döös in that a best fitting surface is found.

We denote by H_s the height of the surface (quadratic or cubic) which best fits the observations and forecasts over a limited area of the chart where

$$H_s = F(x, y) \qquad \ldots\ (1)$$

F being a quadratic or cubic function of x and y . Then the expression E which we minimise in order to obtain our best fitting surface is given by

$$E = \sum_1^m \left\{ p(H_s - H_o)^2 \right\}_r + T^2 \sum_1^n \left\{ p(\underline{V_s} - \underline{V_o})^2 \right\}_r + \sum_1^l \left\{ q(H_s - H_F)^2 \right\}_r \ldots\ (2)$$

where H_o and $\underline{V_o}$ represent the observed 500 mb. contour height and wind respectively, H_F represents a forecast 500 mb. height, $\underline{V_s}$ is the geostrophic wind obtained from the surface represented by Eq. (1), p and q are weighting factors depending upon the distance of the observation or forecast from the grid point for which interpolation is being made, T^2 is a weighting factor relating the fitting of the wind observations to the fitting of the height observations and m , n and l denote the number of height observations, wind observations and forecast heights used to determine F .

One of the main purposes of the work described in this paper was to decide which values of p , q and T^2 gave the best results and whether the use of a cubic form of F gave any improvement over a quadratic form. If we represent the polynomial F (x, y) by

$$F = \sum \alpha_{jk} x^j y^k \qquad \ldots\ (3)$$

where $0 \leq j \leq 2$ and $0 \leq k \leq 2$ in the quadratic case and $0 \leq j \leq 3$ and $0 \leq k \leq 3$ in the cubic case, we determine α_{jk} by solving the set of simultaneous equations

$$\frac{\partial E}{\partial \alpha_{jk}} = 0 \qquad \ldots\ (4)$$

In order to determine the 500 mb. height at each grid point, the data were searched and the six nearest observing stations, at which either a 500 mb. height or wind or both were reported, were found. Only those stations within a radius of 1000 miles of the grid point were used, and sometimes the number of stations chosen was less than six; indeed on rare occasions there could be no stations available. The information available from these stations therefore consisted of m contour heights and n winds where m and n are both integers and $0 \leq m \leq 6$ and $0 \leq n \leq 6$. When m or n is equal to zero then the appropriate summation term in Eq. (1) is of course zero.

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The forecast heights used in this experiment were read from the forecast 500 mb. charts prepared by the Forecast Division at Dunstable, and they were 21 hour forecasts issued for a time three hours previous to the time for which the objective analysis was made. No correction was made for this discrepancy. If this technique was being applied on a routine basis twice daily then numerical forecasts would be available for this purpose and would be based on information only 12 hours old. For the purpose of evaluating the final term in Eq. (2) we took $\ell = 9$, i.e. the summation was effected over the nine nearest grid points to the grid point in question. Thus in the quadratic case we have enough information to determine F at a grid point even if no observations are available. This is not so in the cubic case, and in these circumstances we simply make $H_S = H_F$ at the grid point.

The values of p and T^2 were chosen as a result of some work by Johnson (1956). Two expressions for p were used, and these are shown in Fig. 1 as functions of the distance from the grid point at which interpolation was being carried out. In the first trials a value of T^2 equal to 2 sec.^{2} was used for the quadratic surface and a value of T^2 equal to 16 sec.^{2} was used for the cubic surface. Later, T^2 = 16 sec.^{2} was used for the quadratic surface. Two expressions for q were tried; $q = p/4$ and $q = p/16$. On the Ferranti Mark I machine at Manchester University the computation took 40 sec. per point for the quadratic case and 60 sec. per point for the cubic case.

3. The experiment

The three situations used for the experiment were those for 1500 G.M.T. on the 12th September, 1955, 15th October, 1955 and 31st October, 1955. These situations were chosen because the 500 mb. prontours for 1200 G.M.T. on those days were substantially in error over some part of the area. They were not chosen as being typical of everyday differences between forecast and actual charts, but in this experiment it was necessary that there should be differences between the forecast and the actual observations in order to assess the effect that each was having on the objective analysis.

The objective analyses were made for the 18 x 14 grid used for previous work at Dunstable, but the computations were done firstly for the European half of the grid and secondly for the Atlantic half. This allowed the technique to be tried first for the European network of stations, which is very much denser than the Atlantic network. In order that the reader may assess the smoothness of the results for himself, the results for each grid point have been plotted on the diagrams to the nearest ten feet with the ten thousand figure omitted.

Since the effect of varying the parameters was very similar in each case, only one case is described in detail (Figs. 2a-2h) but the results for the best values of the parameters for the quadratic surface and the results for the cubic surface are given in each case. The observations used in the computations are shown in Figs. 2a, 3a and 4a, together with the analysis of the Forecast Division at Dunstable*. Figs. 2b, 3b, and 4b show the forecast charts which were also used in the experiments.

The first objectively analysed chart (1500 G.M.T. 12th September, 1955) which was computed using this technique is shown in Fig. 2c. T^2 was taken equal to 2 sec.^{2}, $p = p_1 = 2^7(1 + 2^{-29} \cdot 10^8 r^{-4})^{-1}$ where r is the distance in grid lengths of the reporting station from the

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* Certain observations outside the area covered by the background map were also used.

grid point and $q = q_1 = p_1 / 4$. From this and similar computations using the other two sets of data it was obvious that the forecast heights were being given too much weight in all areas except where the network of reporting stations was dense. The objective analyses were therefore repeated using $q = q_2 = p_1 / 16$ and the result of this computation for 1500 G.M.T. 12th September, 1955 is shown in Fig. 2d. In general there was some improvement over Europe in areas of few reporting stations where the forecast was different from the observations. In the analyses for 1500 G.M.T. 12th September, 1955 this improvement is confined to the Mediterranean where the cut-off low is further west in Fig. 2d than in Fig. 2c, which is more in accordance with the observations (Fig. 2a) than the forecast position (Fig. 2b). Over the Atlantic there is also a change away from the forecast values towards the conventional analysis. However, it is easy to see from Fig. 2d that the objective analysis still leaves a lot to be desired over the Atlantic. The main cause of the unsatisfactory nature of these objective analyses over the Atlantic is the introduction of short wavelength features to accommodate isolated observations which are different from the smooth forecast values. The remainder of the experiment was therefore devoted to endeavouring to find ways of removing these short wavelength features from the analyses.

The first step was to repeat the computations using $p = p_2 = 2^7(1 + 2^{-33}.10^8 r^4)^{-1}$. p_2 represents a weighting factor which decreases less rapidly with distance than p_1, and thus each observation has an influence over a greater area than when p_1 was used. The results are shown in Fig. 2e for 12th September, 1955, and the appearance of this chart is typical of the results obtained from the other two situations. Over Europe, only very slight changes occurred and in this area the analyses are as good as can be expected. Over the Atlantic, Fig. 2e shows a marked improvement over Fig. 2d, but there is still some distortion of the main flow in the vicinity of the ship observations. The computations over the Atlantic were repeated taking $T^2 = 16$ sec.^{2} which has the effect of increasing the weight of the wind observations relative to the observed and forecast height. Fig. 2g is typical of the other two computations and it is evident by inspection that this does give a smooth pattern over the Atlantic. Computations were also made in which no forecast heights were used at all (Fig. 2f) but the results were not as good as those obtained in Fig. 2g. At one or two points south of the Azores it was not possible to solve Eqs. (4), due to lack of sufficient data.

One set of computations for each of the three situations was completed using a cubic rather than a quadratic surface, $T^2 = 16$ sec.^{2}, $p = p_2$ and $q = q_2$. The results are shown in Figs. 2h, 3d and 4d, but it does not appear that any significant improvements have been achieved over Figs. 2g (and European half of 2e), 3c and 4c, which represent the best results achieved with the quadratic surface.

4. Conclusion

As a result of this experiment it is evident that for the three situations which were considered, the best objective analysis using a quadratic surface can be obtained when $T^2 = 16$ sec.^{2}, $p = p_2$ and $q = q_2$, although over Europe $T^2 = 2$ sec.^{2} is satisfactory. There does not appear to be any advantage in increasing the length and complexity of the computations in order to use a cubic surface. Indeed, in one or two areas it appears that the quadratic surface probably gives a better answer than the cubic surface as it introduces a greater measure of smoothing. The cubic surface with its extra degrees of freedom is able to fit more closely both the observations and forecast values, and this is not necessarily a good thing in regions not near a reporting station.

It must be remembered that the comparison between analyses has been subjective in this experiment, but an inspection of the actual results plotted on the diagrams indicates that the contour lines could hardly have been drawn

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differently. Whether the objective analysis technique suggested in this report can produce charts sufficiently smooth and accurate to be used as initial data for time integrations remains to be tested, but visual inspection of the charts suggests that this will be possible. It is certain that this technique used on an operational basis with a Ferranti Mark II computer would enable the initial data for a time integration to be available inside the machine considerably sooner than if the data had first to be plotted, the chart analysed, values read off at grid points, and these values punched on to paper tape.

5. Acknowledgement

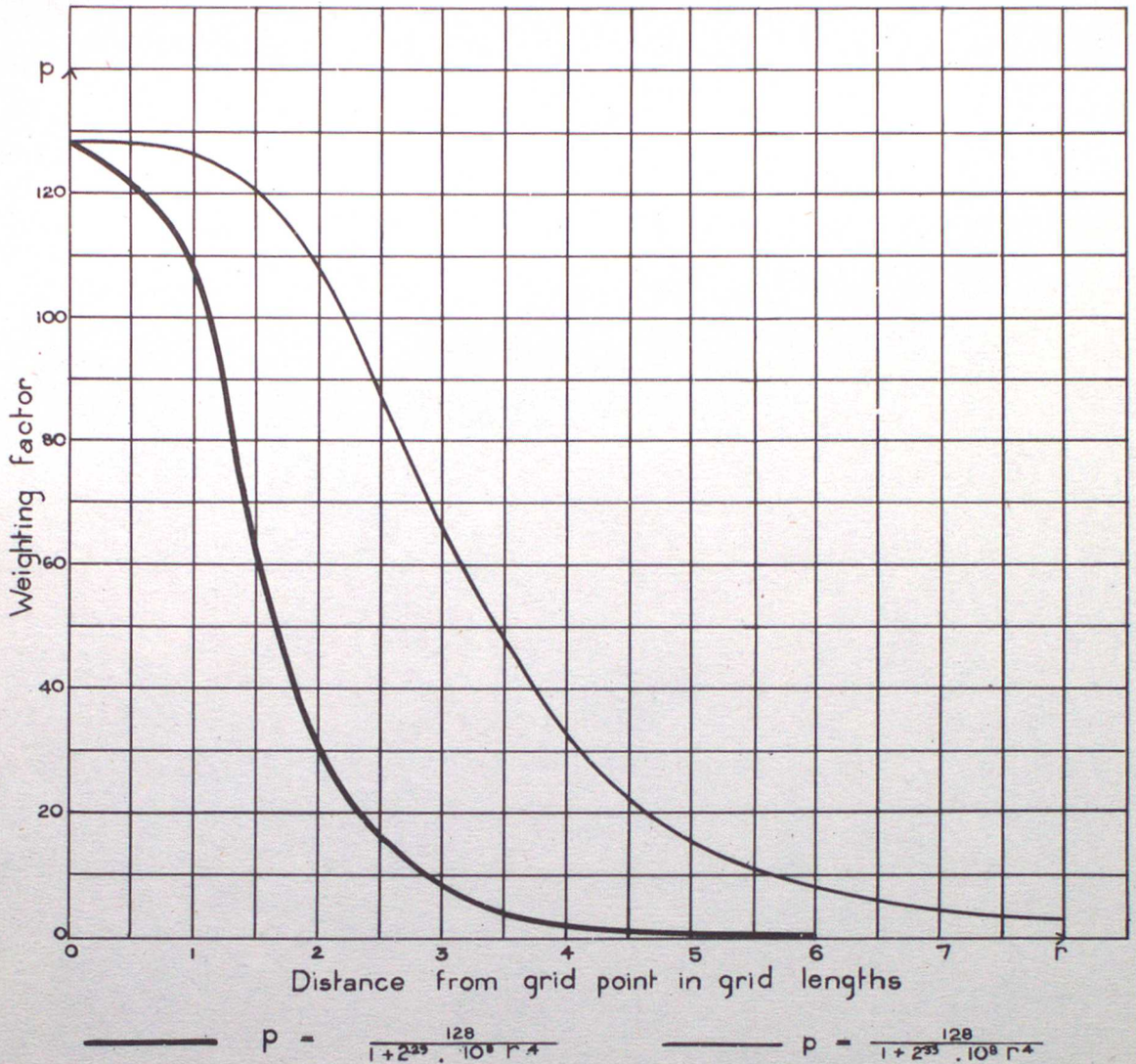
I wish to acknowledge the fact that Miss Huckle was responsible for a large part of the programming of the computations connected with this work.

References

B. Gilchrist and C.P. Cressman — Tellus, Vol. 6, No. 4, 1954, p.309.

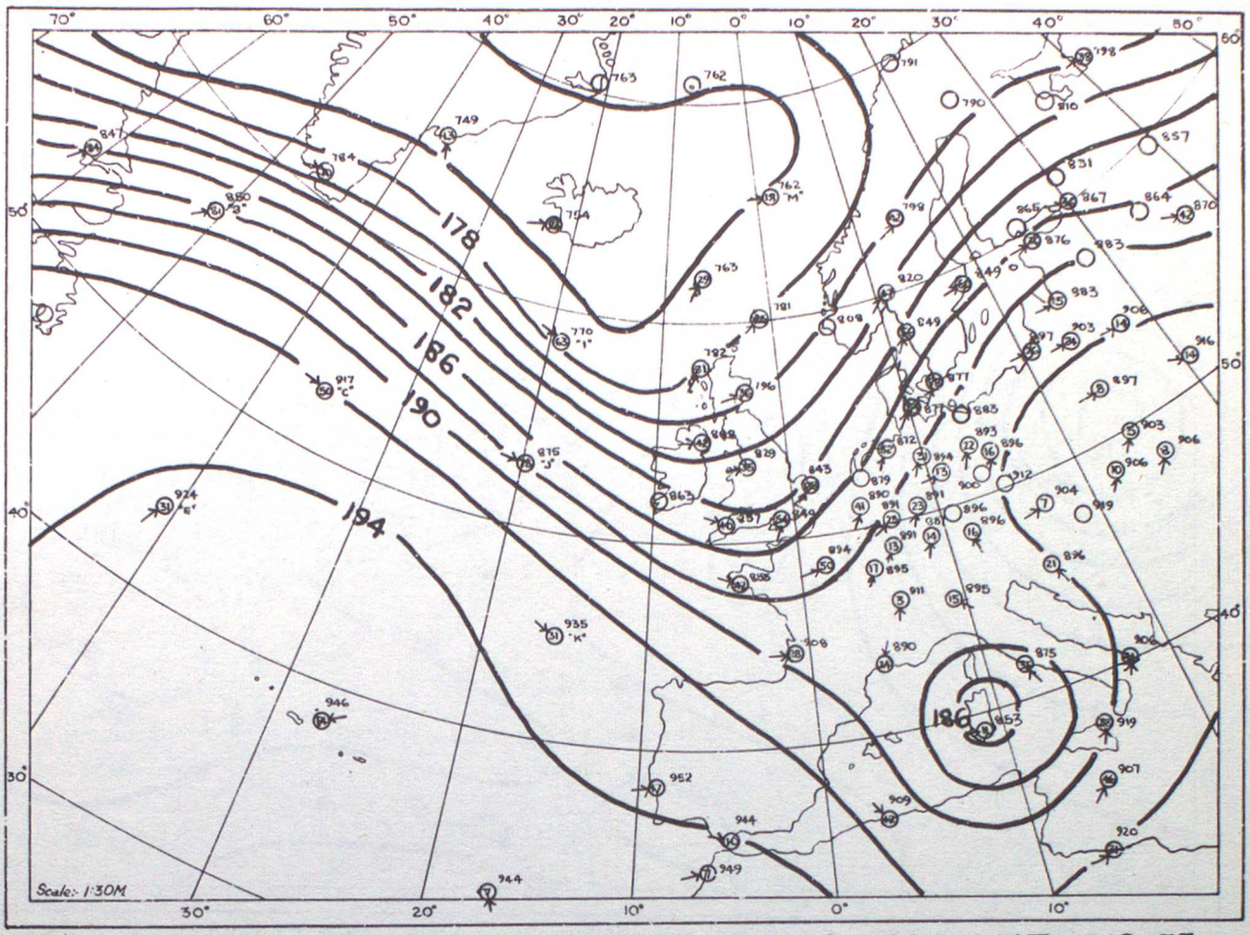
P. Bergthorrson and B. Döös — Tellus, Vol. 7, No. 3, 1955, p.329.

D.H. Johnson — Meteorological Research Papers, Air Ministry, London, No. 965, 1956.



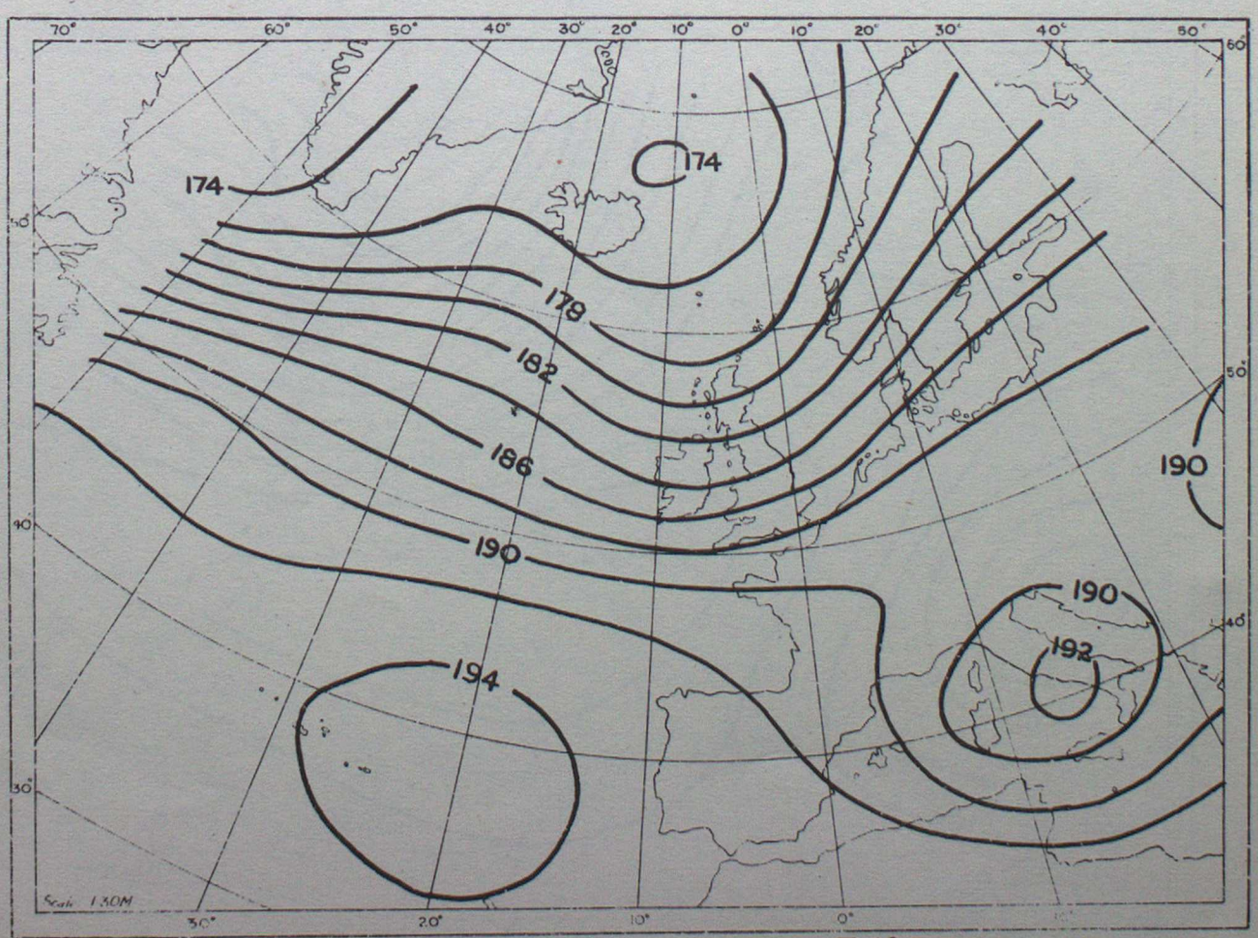
Distance weighting factors

Fig. 1



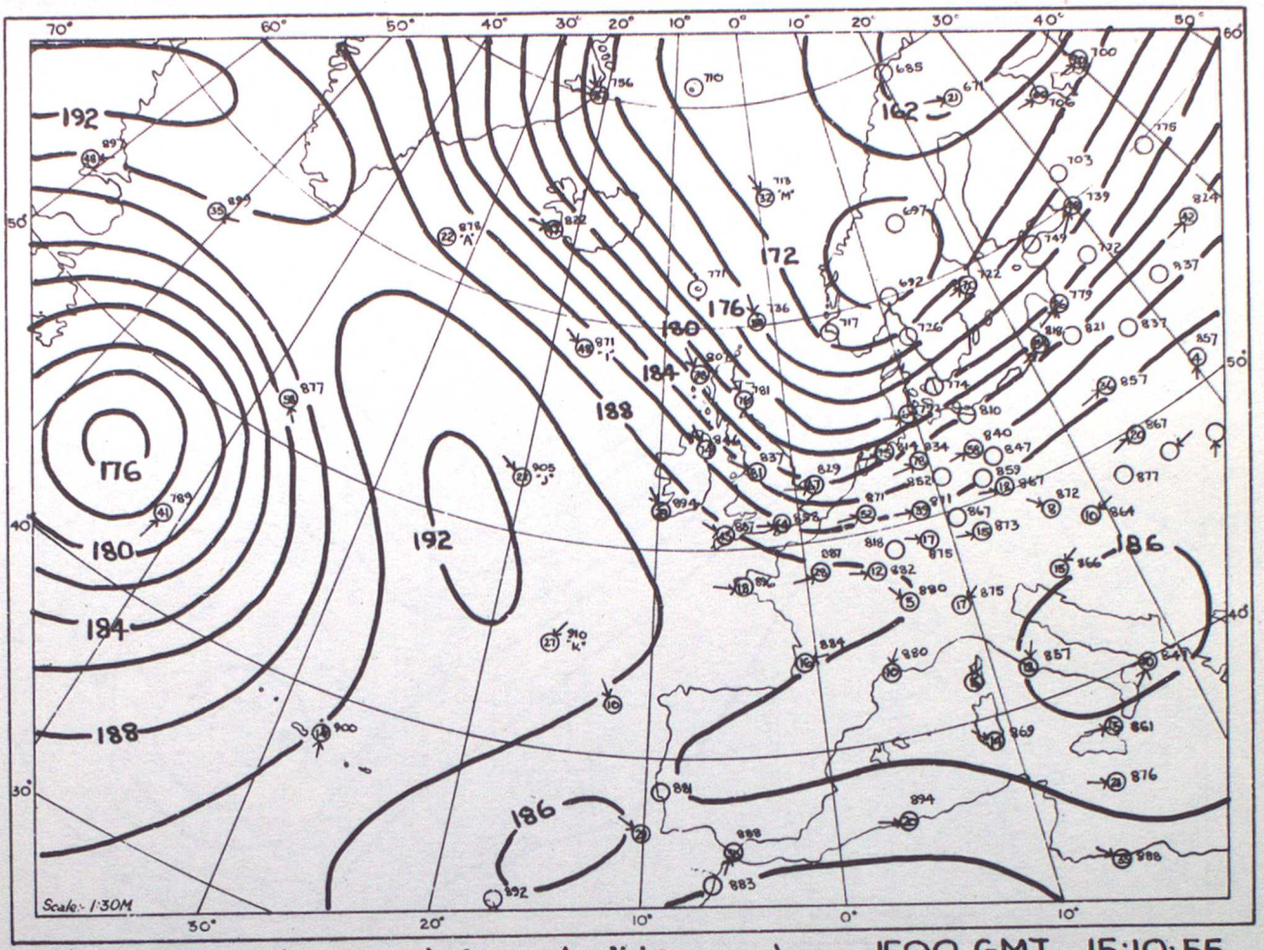
500mb. Observations and forecast division analysis for 1500 G.M.T. 12:9:55

Fig. 2(a)



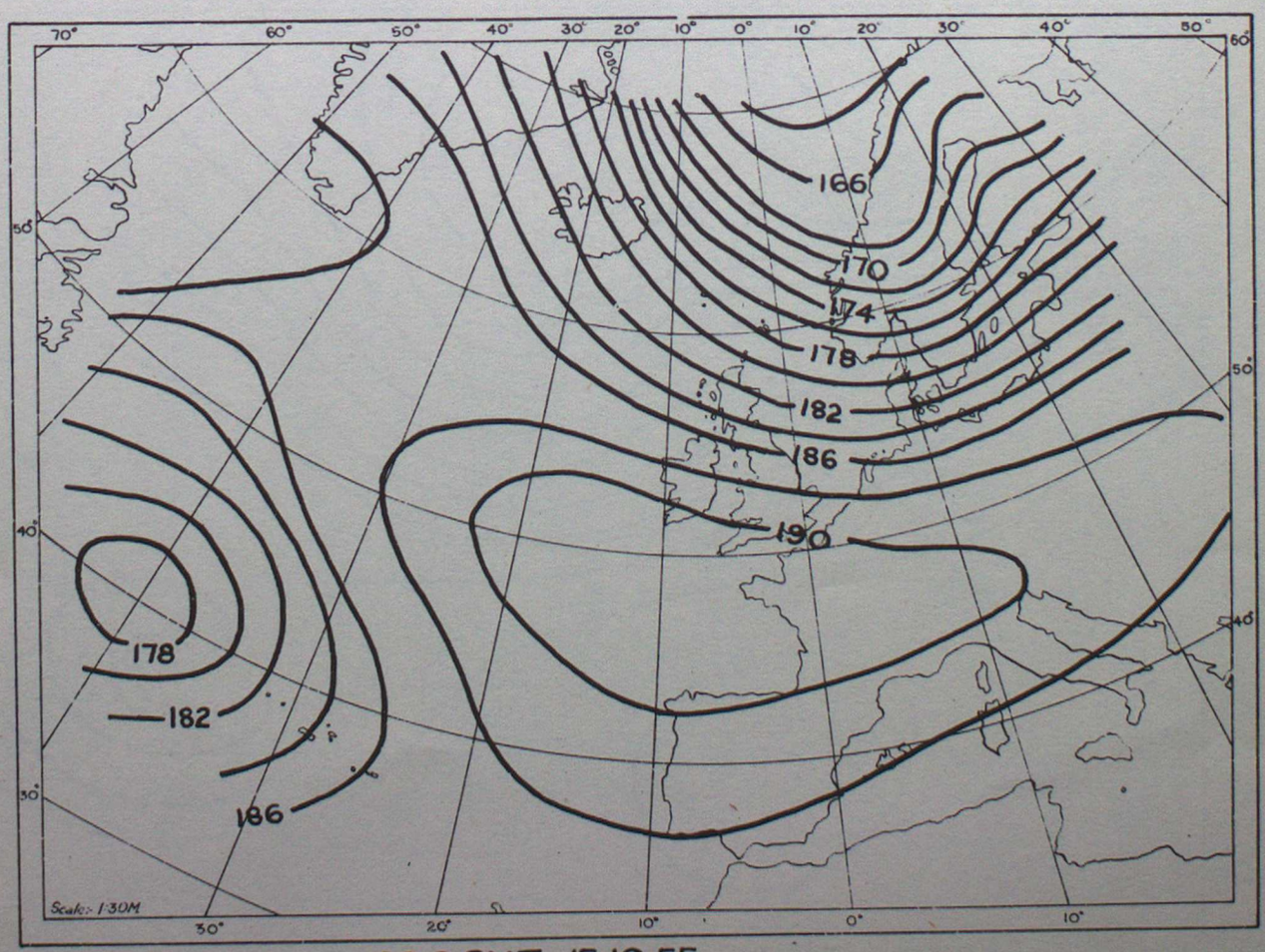
500mb prontour 1500 G.M.T. 12:9:55

Fig 2(b)



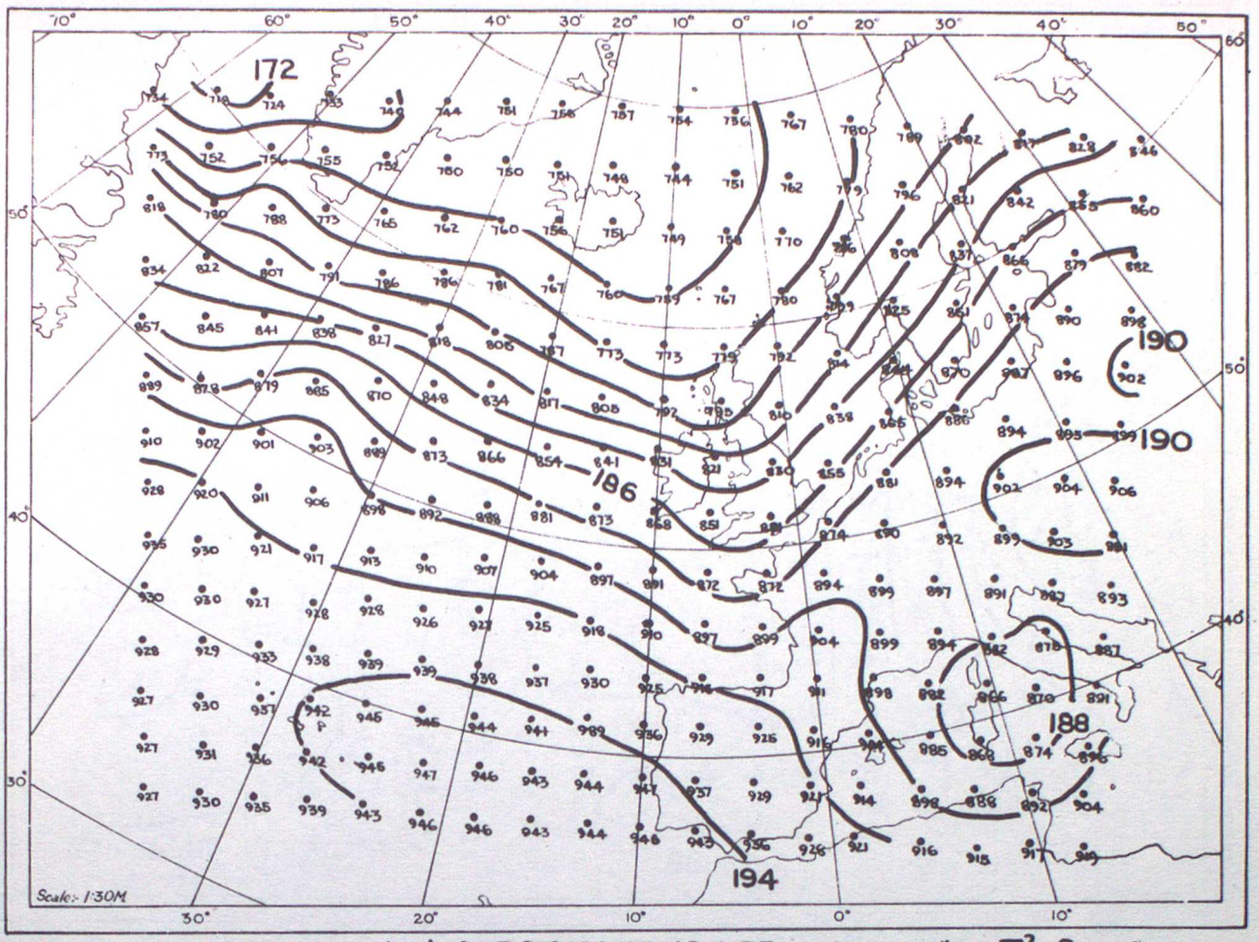
500mb. Observations and forecast division analysis 1500 G.M.T. 15:10:55

Fig 3(a)



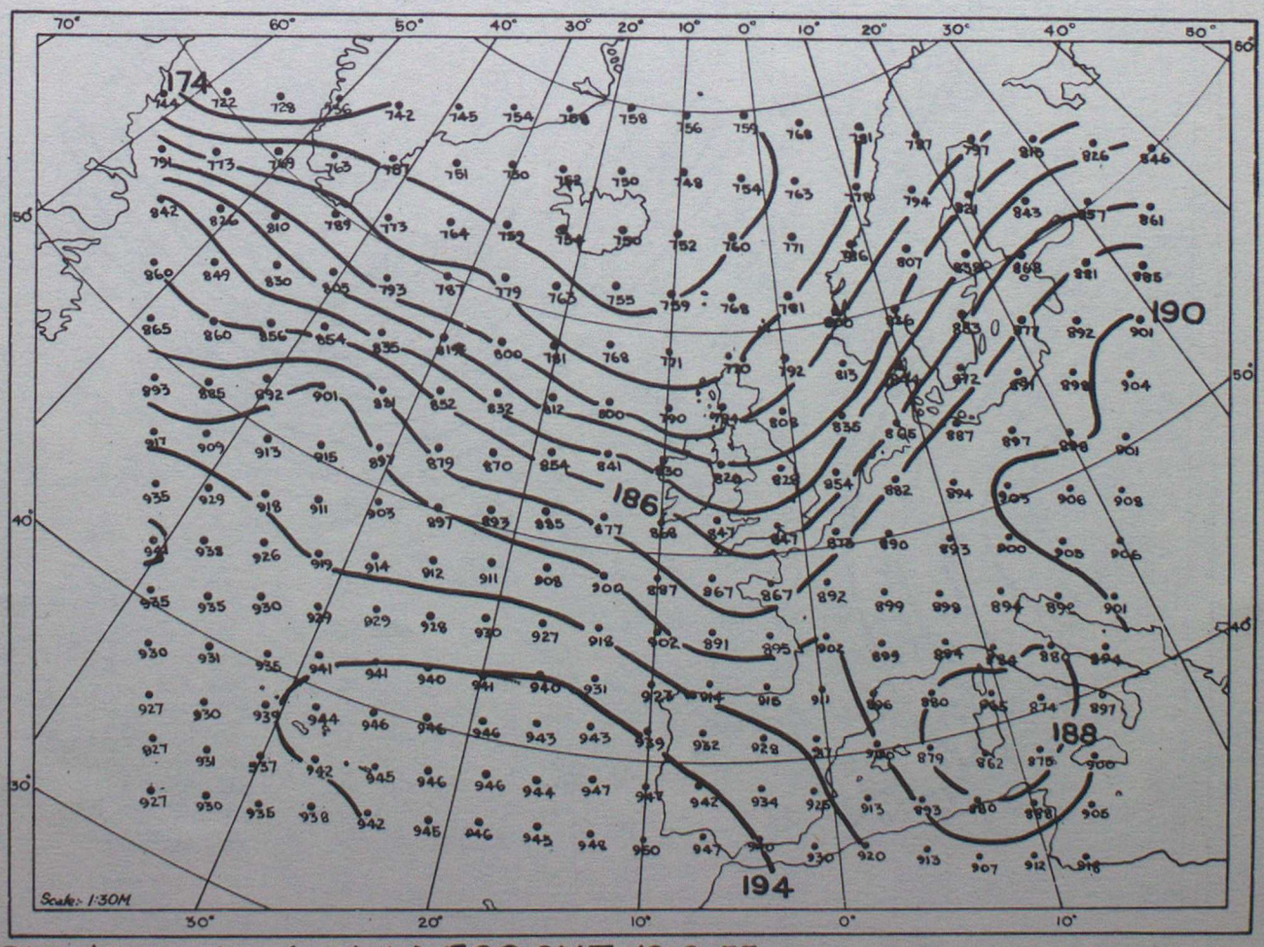
500 mb prontour, 1200 G.M.T. 15:10:55

Fig. 3(b)



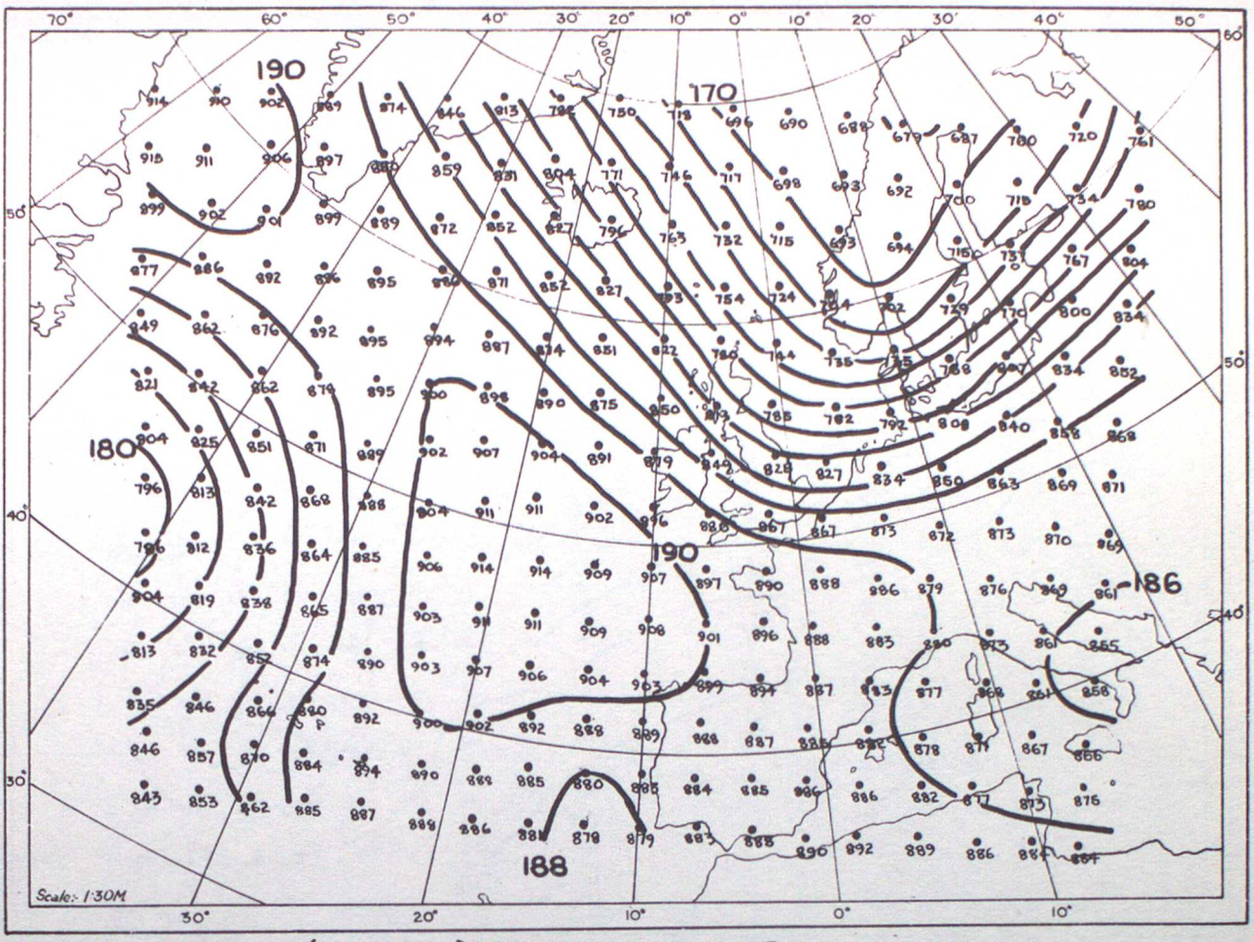
Objective analysis (quadratic) 1500 G.M.T. 12:9:55 $p=p_1$ $q=\frac{1}{4}p_1$ $T^2=2\,sec^2$

Fig. 2 (c)



Objective analysis (quadratic) 1500 G.M.T. 12:9:55
$p=p_1$ $q=\frac{1}{16}p$ $T^2=2\,sec^2$

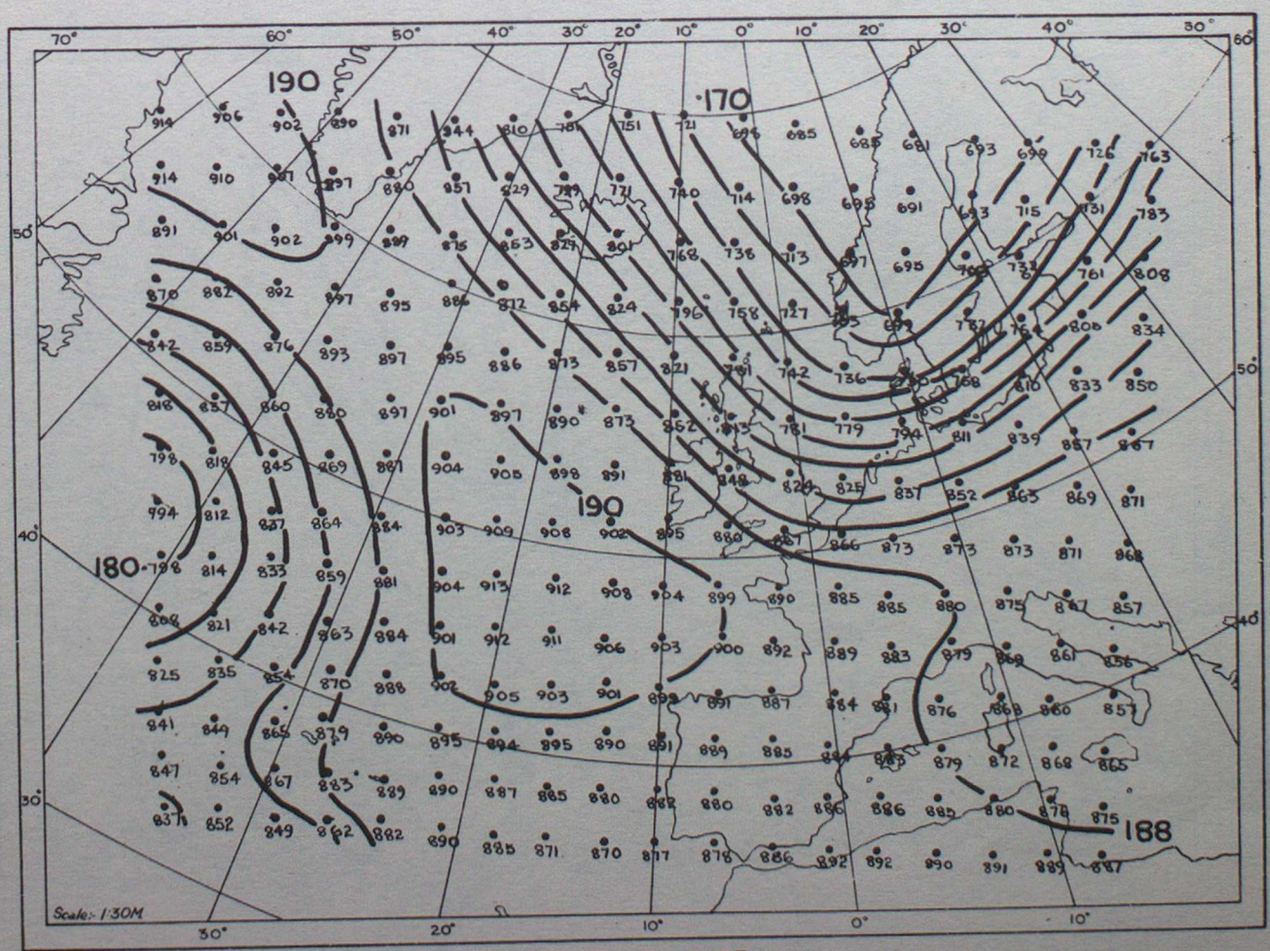
Fig. 2 (d)



Objective analysis (quadratic) 1500 G.M.T. 15:10:55

$P = P_2$ $q = \frac{1}{16} P_2$ $T^2 = 2$ sec² (Europe) $T^2 = 16$ sec² (Atlantic)

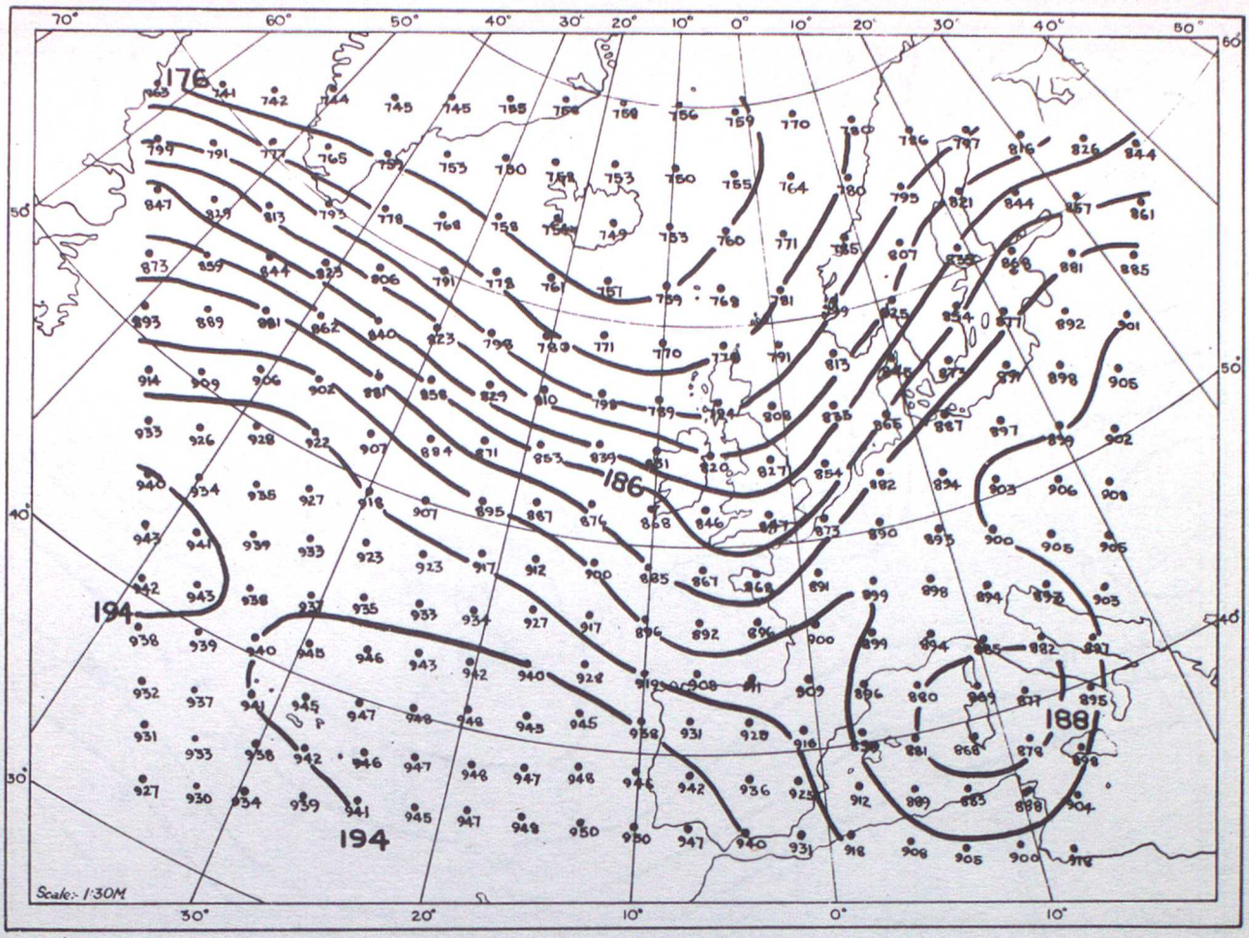
Fig. 3(c)



Objective analysis (cubic) 1500 G.M.T. 15:10:55

$P = P_2$ $q = \frac{1}{16} P_2$ $T^2 = 16$ sec²

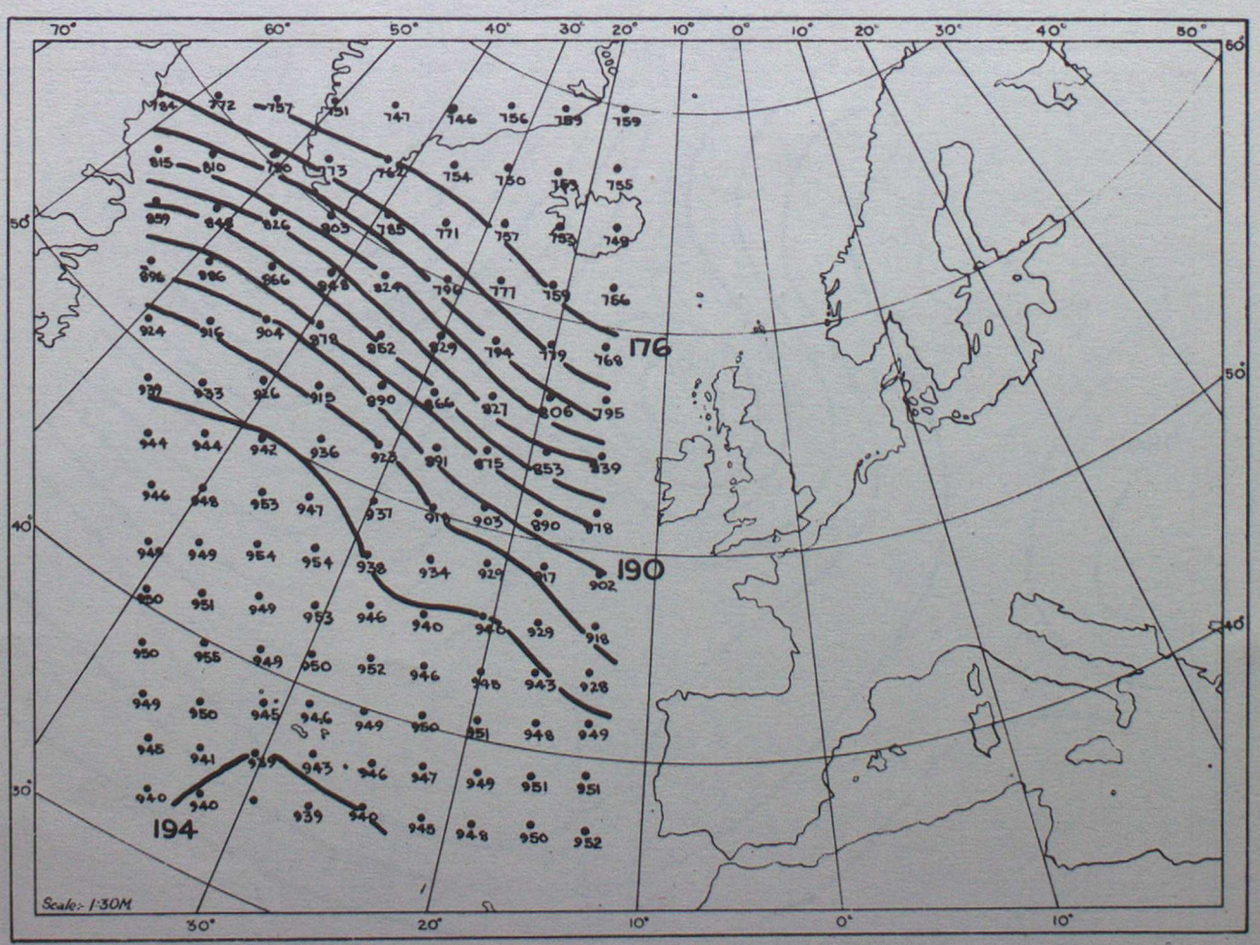
Fig. 3(d)



Objective analysis (quadratic) 1500 G.M.T. 12:9:55

$p = p_2 \quad q = \frac{1}{16} p_2 \quad T^2 = 2\,\text{sec}^2$

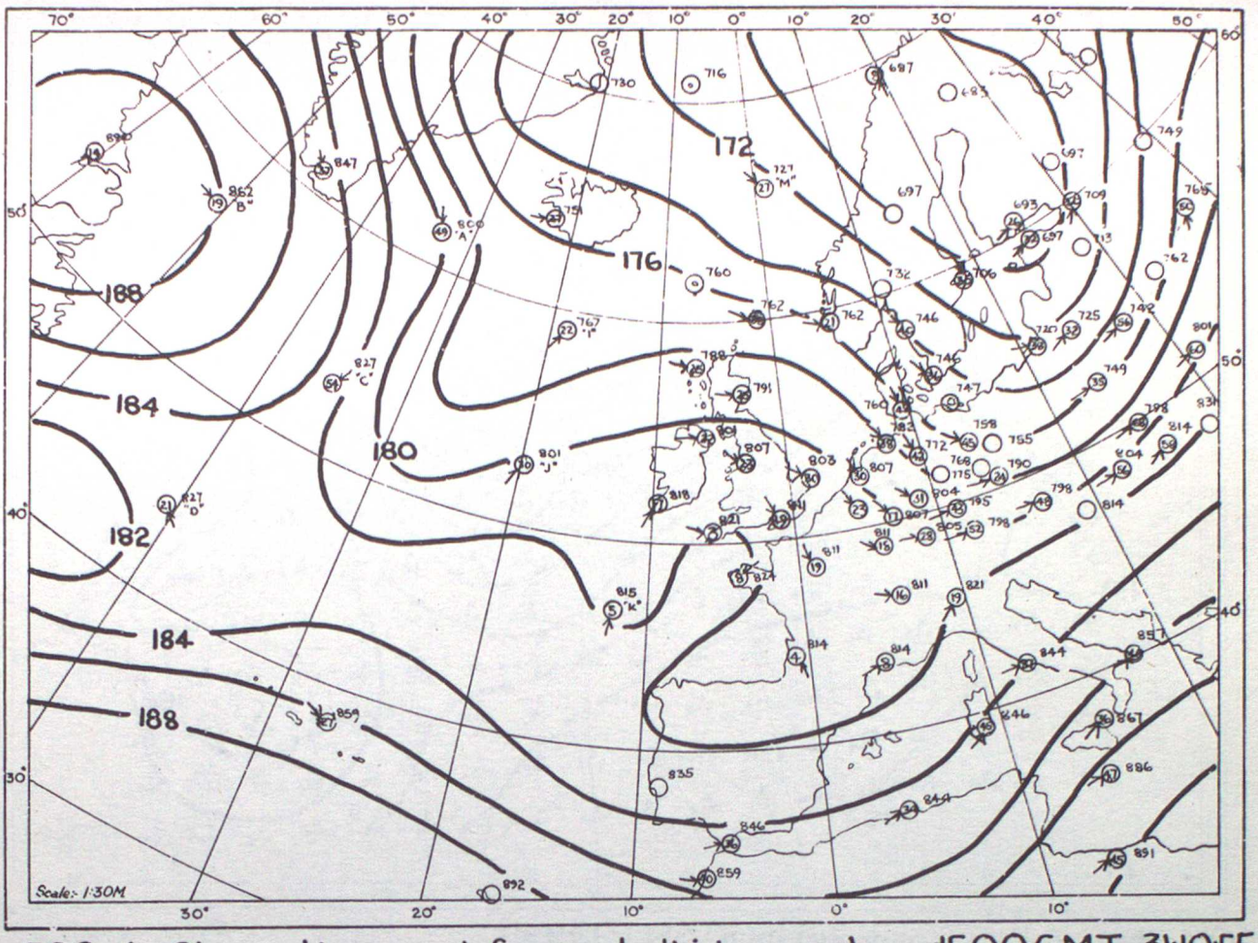
Fig. 2 (e)



Objective analysis (quadratic) 1500 G.M.T. 12:9:55

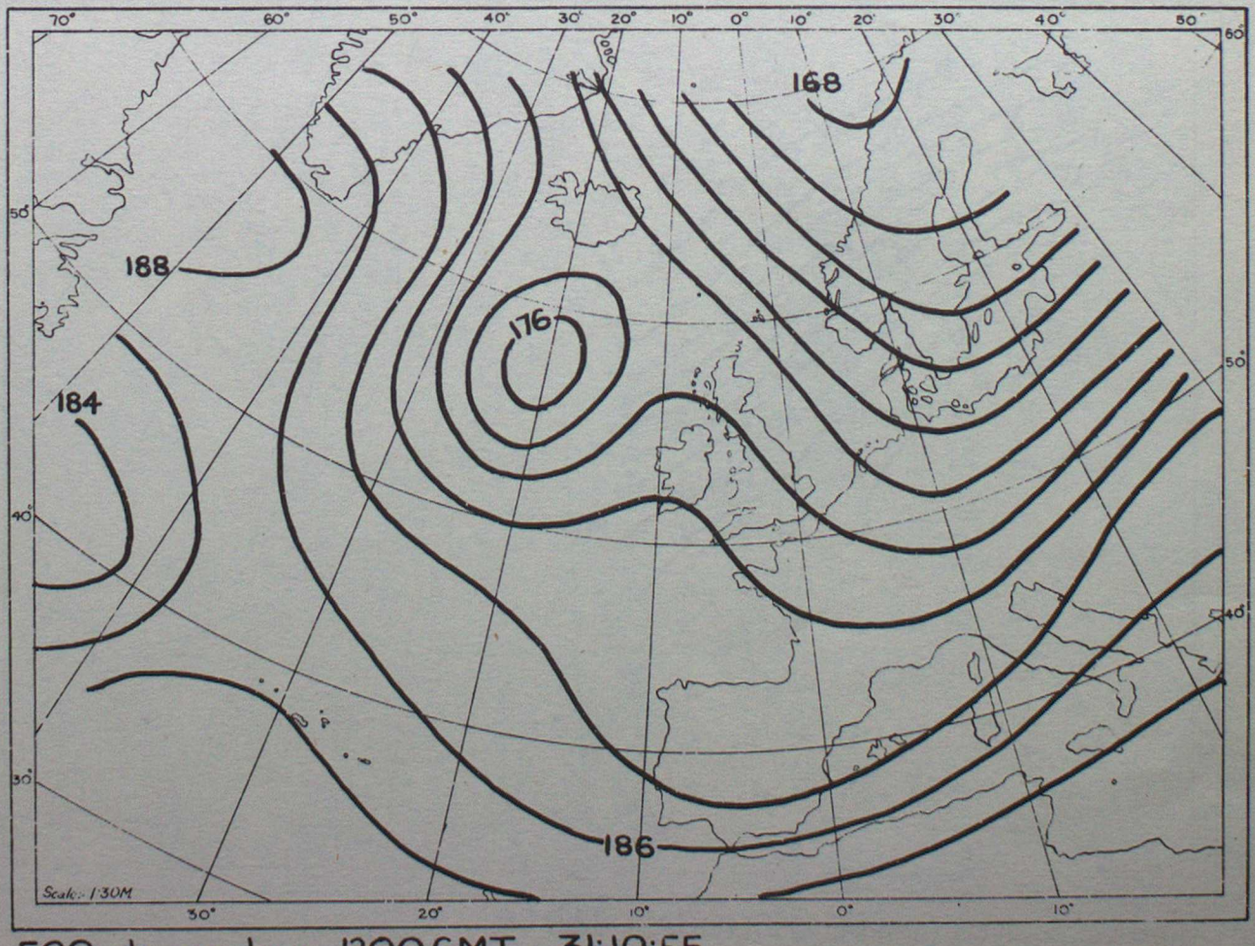
$p = p_2 \quad q = 0 \quad T^2 = 2\,\text{sec}^2$

Fig 2 (f)



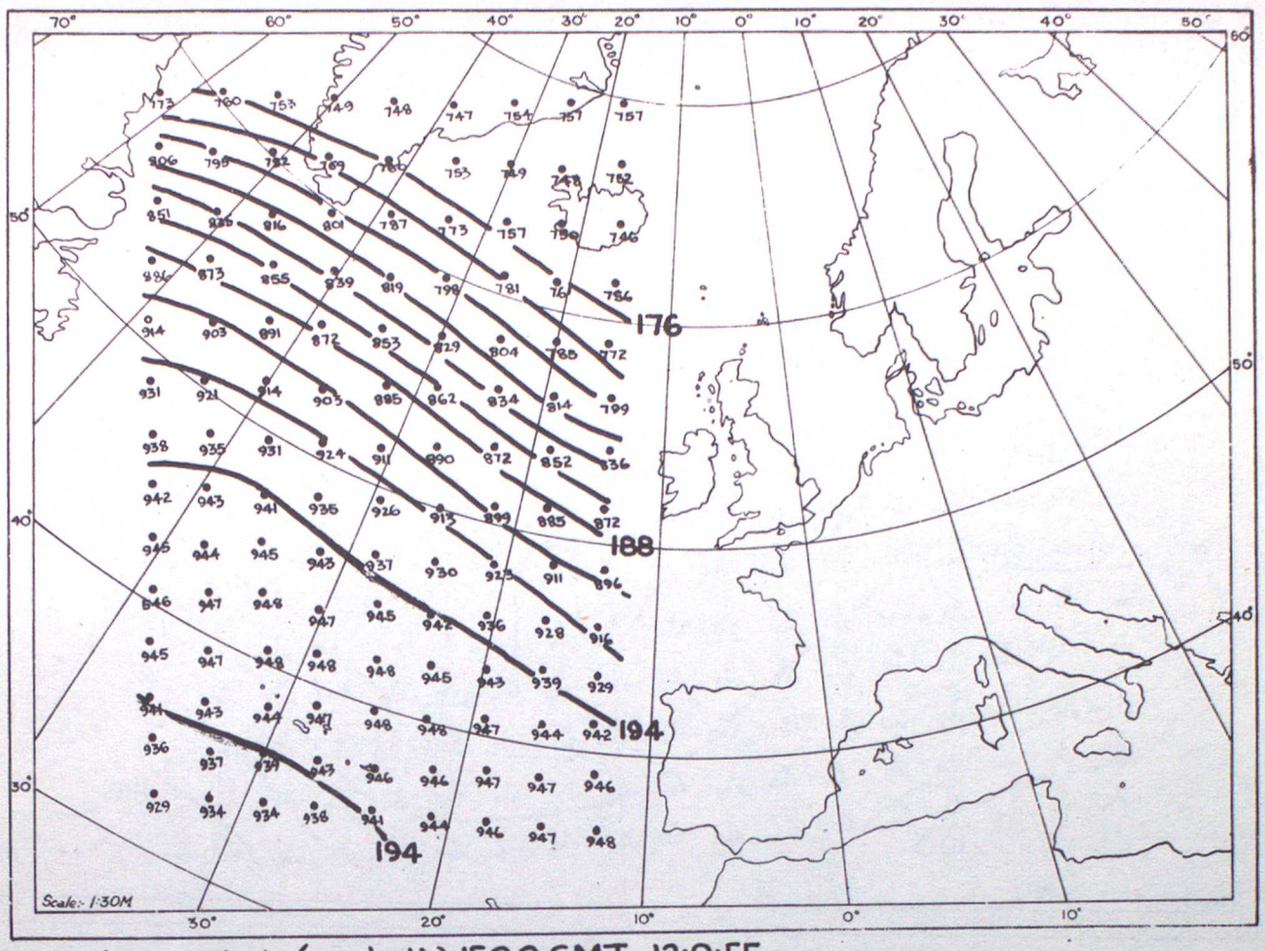
500mb Observations and forecast division analysis 1500 G.M.T. 31:10:55

Fig 4(a)



500mb. prontour 1200 G.M.T. 31:10:55

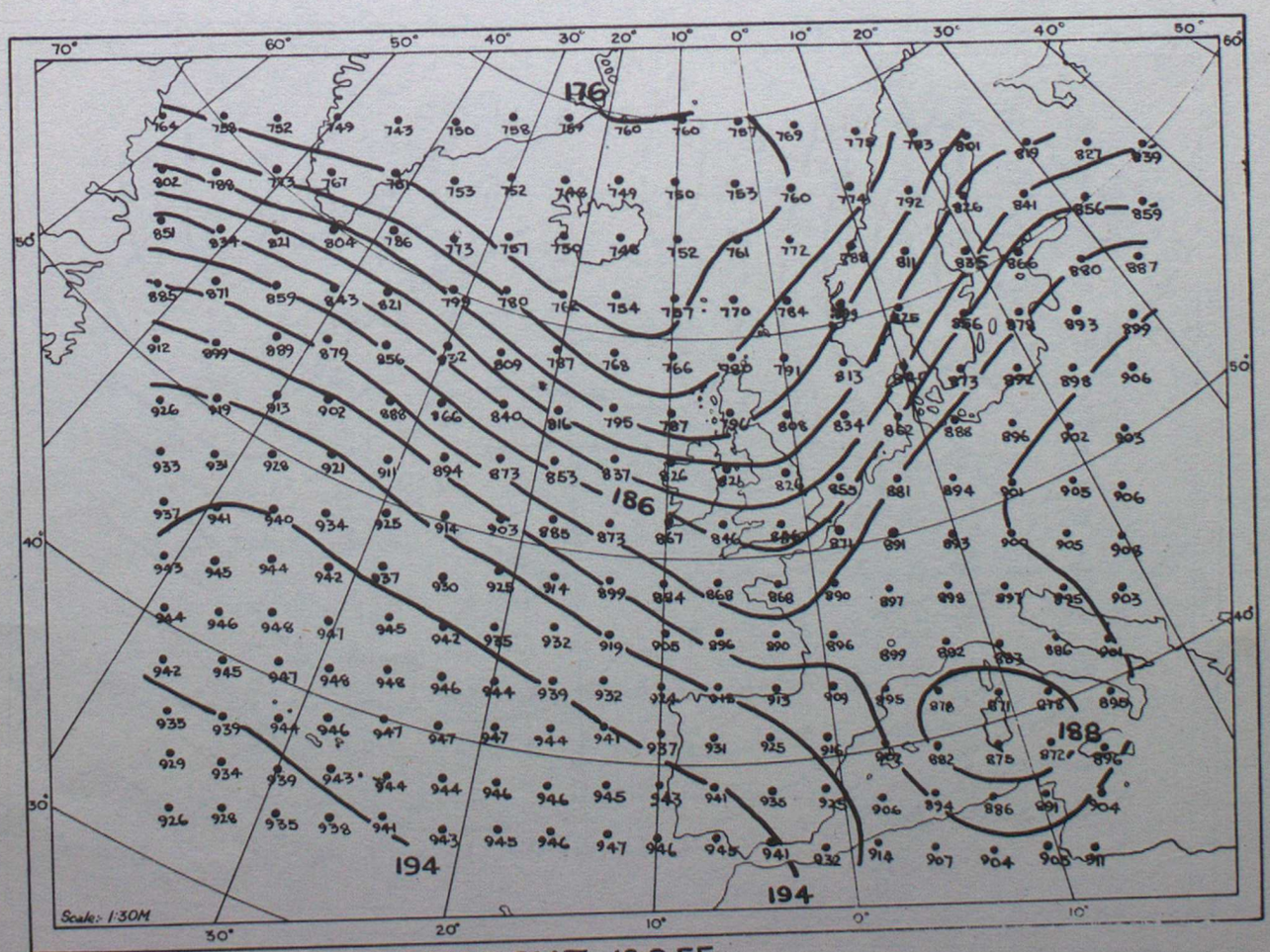
Fig. 4 (b)



Objective analysis (quadratic) 1500 G.M.T. 12:9:55

$P = P_2$ $q = \frac{1}{16} P_2$ $T^2 = 16\,\text{sec}^2$

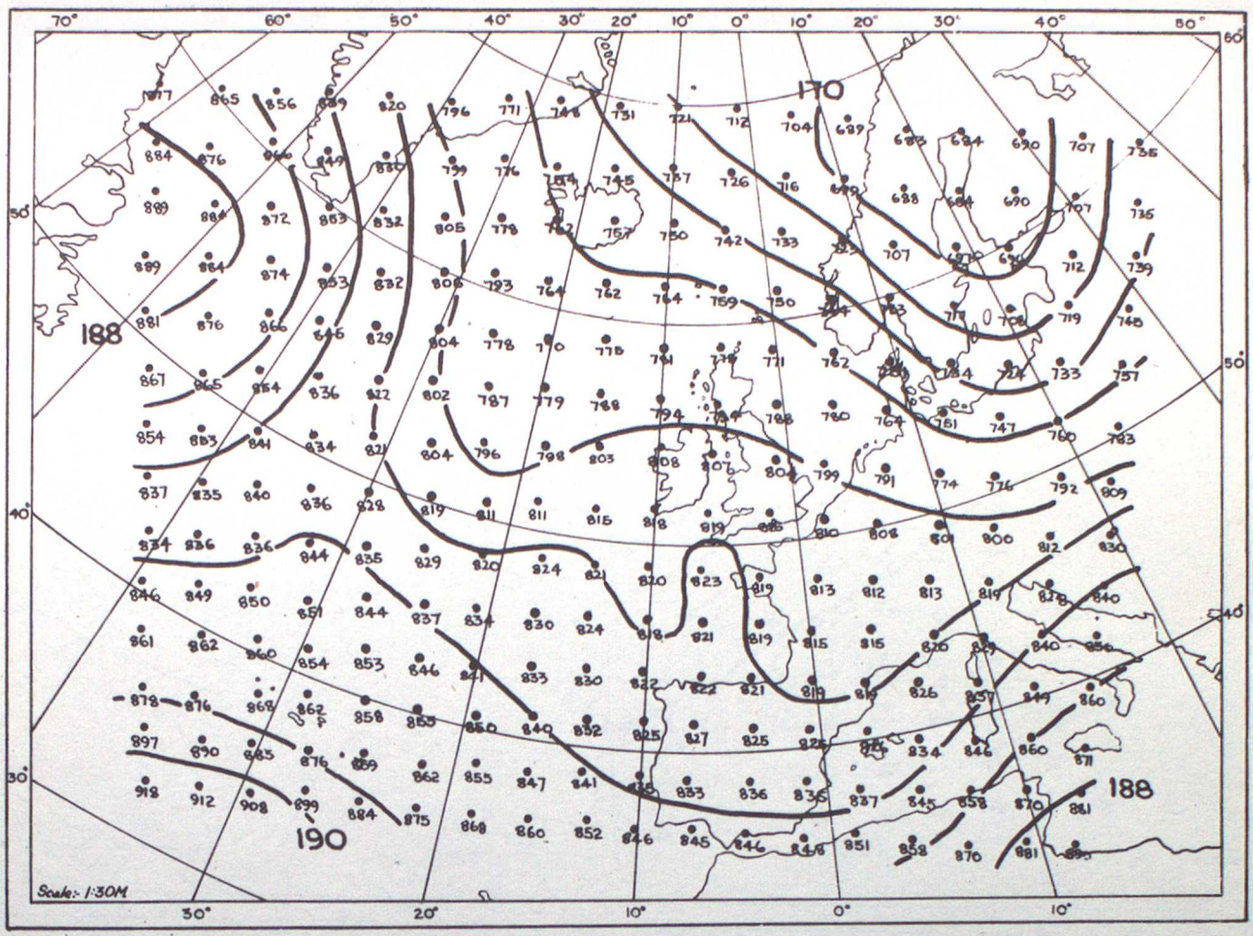
Fig. 2 (g)



Objective analysis (cubic) 1500 G.M.T. 12:9:55

$P = P_2$ $q = \frac{1}{16} P_2$ $T^2 = 16\,\text{sec}^2$

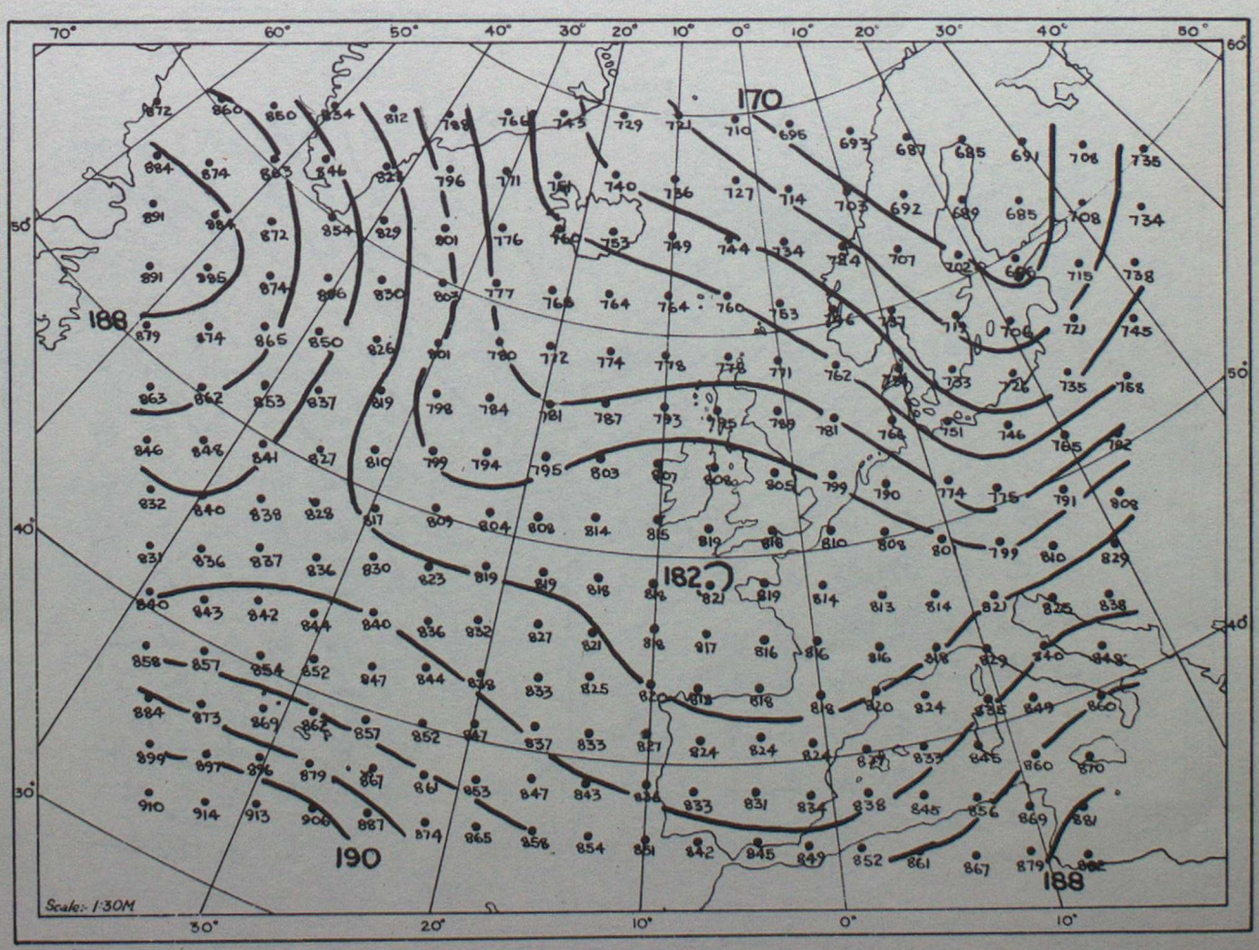
Fig. 2 (h)



Objective analysis (quadratic) 1500 G.M.T. 31:10:55

$P = P_2$ $q = \frac{1}{16} P_2$ $T^2 = 2\,\text{sec}^2$ (Europe) $T^2 = 16\,\text{sec}^2$ (Atlantic)

Fig. 4 (c)



Objective analysis (cubic) 1500 G.M.T. 31:10:55

$P = P_2$ $q = \frac{1}{16} P_2$ $T^2 = 16\,\text{sec}^2$

Fig. 4 (d)